Chapter 6
Annual Worth Analysis

Lecture slides to accompany
Engineering Economy
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LEARNING OUTCOMES

1. Advantages of AW
2. Capital Recovery and AW values
3. AW analysis
4. Perpetual life
5. Life-Cycle Cost analysis
Advantages of AW Analysis

AW calculated for only one life cycle

Assumptions:

- Services needed for \textit{at least the LCM} of lives of alternatives
- Selected alternative \textit{will be repeated} in succeeding life cycles in same manner as for the first life cycle
- All cash flows \textit{will be same} in every life cycle (i.e., will change by only inflation or deflation rate)
Alternatives usually have the following cash flow estimates

- **Initial investment, P** – First cost of an asset
- **Salvage value, S** – Estimated value of asset at end of useful life
- **Annual amount, A** – Cash flows associated with asset, such as annual operating cost (AOC), etc.

### Relationship between AW, PW and FW

\[
AW = PW(A/P, i\%, n) = FW(A/F, i\%, n)
\]

\(n\) is years for equal-service comparison (value of LCM or specified study period)
Calculation of Annual Worth

AW for one life cycle is the same for all life cycles!!

An asset has a first cost of $20,000, an annual operating cost of $8000 and a salvage value of $5000 after 3 years. Calculate the AW for one and two life cycles at \( i = 10\% \)

\[
AW_{\text{one}} = - 20,000 \times (A/P, 10\%, 3) - 8000 + 5000 \times (A/F, 10\%, 3)
\]
\[
= -$14,532
\]

\[
AW_{\text{two}} = - 20,000 \times (A/P, 10\%, 6) - 8000 - 15,000 \times (P/F, 10\%, 3) \times (A/P, 10\%, 6)
+ 5000 \times (A/F, 10\%, 6)
\]
\[
= -$14,532
\]
Capital Recovery and AW

Capital recovery (CR) is the equivalent annual amount that an asset, process, or system must earn each year to just recover the first cost and a stated rate of return over its expected life. Salvage value is considered when calculating CR.

\[ CR = -P(A/P, i\%, n) + S(A/F, i\%, n) \]

Use previous example: (note: AOC not included in CR)

\[ CR = -20,000(A/P, 10\%, 3) + 5000(A/F, 10\%, 3) = $ - 6532 \text{ per year} \]

Now

\[ AW = CR + A \]

\[ AW = - 6532 - 8000 = $ - 14,532 \]
Selection Guidelines for AW Analysis

One alternative: If $AW \geq 0$, the requested MARR is met or exceeded and the alternative is economically justified.

Two or more alternatives: Select the alternative with the AW that is **numerically largest**, that is, less negative or more positive. This indicates a lower AW of cost for cost alternatives or a larger AW of net cash flows for revenue alternatives.
A company is considering two machines. Machine X has a first cost of $30,000, AOC of $18,000, and S of $7000 after 4 years. Machine Y will cost $50,000 with an AOC of $16,000 and S of $9000 after 6 years. Which machine should the company select at an interest rate of 12% per year?

**Solution:**

\[
AW_X = -30,000(A/P, 12\%, 4) - 18,000 + 7,000(A/F, 12\%, 4) \\
= -$26,412
\]

\[
AW_Y = -50,000(A/P, 12\%, 6) - 16,000 + 9,000(A/F, 12\%, 6) \\
= -$27,052
\]

Select Machine X; it has the numerically larger AW value.
**AW of Permanent Investment**

**Solution:** Find AW of C over 5 years and AW of D using relation $A = Pi$

\[
\text{AW}_C = -50,000\left(\frac{A}{P, 10\%, 5}\right) - 20,000 + 5,000\left(\frac{A}{F, 10\%, 5}\right) \\
= -$32,371
\]

\[
\text{AW}_D = Pi + \text{AOC} = -250,000(0.10) - 9,000 \\
= -$34,000
\]

Select alternative C

<table>
<thead>
<tr>
<th></th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Cost, $</td>
<td>-50,000</td>
<td>-250,000</td>
</tr>
<tr>
<td>Annual operating cost, $/year</td>
<td>-20,000</td>
<td>-9,000</td>
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<tr>
<td>Salvage value, $</td>
<td>5,000</td>
<td>75,000</td>
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<tr>
<td>Life, years</td>
<td>5</td>
<td>$\infty$</td>
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Typical Life-Cycle Cost Distribution by Phase
Life-Cycle Cost Analysis

LCC analysis includes *all* costs for *entire* life span, from concept to disposal.

Best when large percentage of costs are *M&O*.

Includes phases of *acquisition, operation, & phaseout*.

- Apply the AW method for LCC analysis of 1 or more cost alternatives
- Use PW analysis if there are revenues and other benefits considered
Summary of Important Points

- AW method converts all cash flows to *annual value at MARR*

- Alternatives can be *mutually exclusive, independent, revenue, or cost*

- AW comparison is *only one life cycle of each alternative*

- For infinite life alternatives, annualize *initial cost as $A = P(i)$*

- Life-cycle cost analysis includes *all costs over a project’s life span*